TOWARDS A METRIC FOR AN AUTOMATIC HULL MESH COARSENING STRATEGY

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INTRODUCTION

In numerical simulations of various areas of science and engineering, e.g., biomechanical engineering [1] or ray tracing [2], the discretization of objects in physical space is an integral step. In particular for three-dimensional cases, where a volume mesh generation is required, the mesh generation is rather critical for the robustness, stability, and accuracy for the subsequent simulation steps. One common approach to generate a volume mesh utilizes the Advancing Front method [3, 4]. This approach uses a two-dimensional hull mesh (i.e., a closed surface) as starting point, which potentially consists of an unnecessary large amount of mesh elements. Obviously, the properties of the hull mesh (e.g. the number of triangles and their quality) heavily influence the quality and the execution time of the volume mesh generation process and the subsequent simulations. The aim of this work is to introduce a novel combined comparison metric to compare the quality of the coarsened mesh to its original input. This work paves the way for an automatic domain- and mesh-specific hull mesh coarsening method to aid subsequent volume meshing steps.

COARSENING ALGORITHM

Within this work, CGAL's Triangulated Surface Mesh Simplification [5] has been utilized to coarsen two widely used and representative geometries. The first geometry, the *bunny*, consists originally of 69 451 triangles and the second geometry, the *elephant*, consists originally of 5 558 triangles (see Figure 1). One important parameter which can be set using CGAL's algorithm is the *stop ratio*, denoting the desired number of remaining triangles in the coarsened mesh.

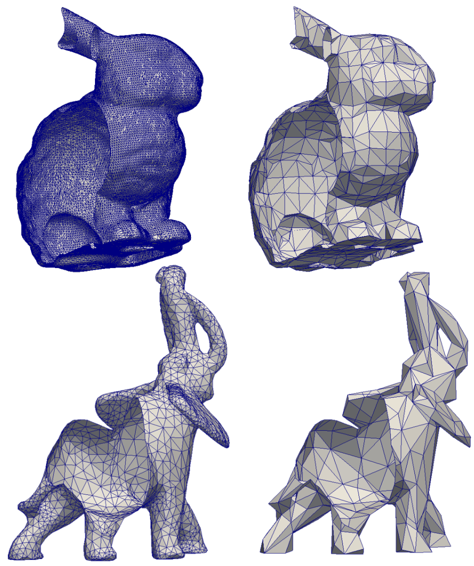

Figure 1: Cross-sections of the original and exemplary coarsened meshes of the two test geometries. The *bunny* in the top row was coarsened from 69 451 to 1 513 triangles, the *elephant* from 5 558 to 554.

DEVISED METRIC

To evaluate the coarsening algorithm, we selected four widely used metrics to capture various aspects of the coarsening process. We picked the triangle shape quality T, the geometric distance to the original mesh H, the differences in curvature C, and the surface area deviation A [6–8]. Using these metrics we devised a combined quality metric M, which maps all aspects to a single scalar-valued metric:

$$M = \alpha(1-T) + \beta H + \gamma C + \delta A, \quad \alpha, \beta, \gamma, \delta \in \mathbb{R}, \quad M \in [0, \infty). \tag{1}$$

Here α, β, γ, and δ denote the weighting factors which have to be determined with respect to the actual application of the considered mesh [1, 2]. Note that two exactly equal meshes yield $M = \alpha/2$, since in this case H, C, and A are equal to 0 and $T = 0.5$.

However, coarsening can lead to enhanced triangle quality, which subsequently results in $M < \alpha/2$.

RESULTS AND DISCUSSION

For coarsening the test geometries, we investigated different stop ratios in the range of $[0.004, 0.75]$. We computed the four different metrics T, H, C, and A for each coarsened mesh. As expected, our studies show that the values for H, C, and A vary up to an order of magnitude of 2 (e.g., $H = 0.00064$ to $C = 0.07377$). Since we wanted to attribute each geometric feature the same level of importance and to preserve various prominent geometric features (i.e., curvatures), we empirically chose the ratio of the weighting factors β, γ, and δ. Due to the fact that a poor quality of one single triangle in the coarse mesh heavily influences not only the volume mesh generation process, but also the subsequent numerical simulations, we had to ensure that the triangle shape metric T was of the same level of importance as the three geometric features. Therefore, we chose its weighting factor α such that the first term in Equation 1 is of the same order of magnitude as the sum of the other three terms. By considering all these demands, we found a suitable choice of absolute weighting factors as shown in Table 1. These factors vary for each geometry, because the difference of each of the four metrics T, H, C, and A between the original and coarse mesh depends on the number of removed triangles which is mesh specific. Additionally, our investigations show that by applying these absolute factors a coarsened mesh can be judged as *good*, if the condition $M < 0.4$ holds. Above this threshold either the triangle shape metric or the geometric feature metrics tend to become unreasonably high.

Model	α	β	γ	δ	M_{equal}	M_{max}
Bunny	0.34	11.5	1.15	0.92	0.17	13.91
Elephant	0.18	8.82	0.88	0.70	0.09	10.58

Table 1: Weighting factors and ranges of M in Equation 1 obtained in this study for the considered geometries. M_{equal} denotes the value for two exactly equal meshes and M_{max} the worst-case maximum.

CONCLUSION

The investigated algorithm together with the presented quality metric is the first step towards devising an automatic hull mesh coarsening work flow. Future work will focus on analyzing the domain-specific weighting factors and on devising an automatic method to compute those factors to ultimately guide a coarsening work flow via the combined scalar-valued quality metric.

ACKNOWLEDGMENT

The financial support by the Austrian Federal Ministry of Science, Research and Economy and the National Foundation for Research, Technology and Development is gratefully acknowledged as is the support by the TU Wien IP project *"Parallel 3D Mesh Generation for Bio-Micro- & Nanoelectromechanical Systems"*.

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