MINIMIZING CROSSINGS IN CONSTRAINED TWO-SIDED CIRCULAR GRAPH LAYOUTS

Fabian Klute[a], Martin Nöllenburg[a]

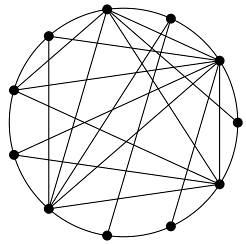
[a]E186/1 - Algorithms and Complexity Group at TU Wien

INTRODUCTION

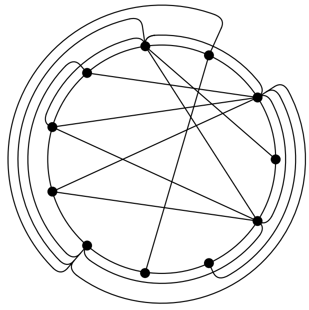
Node-link diagrams are a classical way to visualize data which is represented as a graph. Graph drawing aims to find algorithms which automate the layouting. Of course one needs criteria for what a good layout is. One of the most used ones is the number of crossings between single edges in the drawing. This is not only intuitively a good criteria, but was also confirmed by e.g. Purchase [5].

We focus on a particular drawing style called circular layouts. In such a layout vertices are positioned on a circle, while the edges are drawn as straight-line chords of said circle, see Figure 1a. Finding a vertex order that minimizes the crossings is NP-hard [3] making it unlikely that an exact and efficient algorithm exists. Heuristics and approximation algorithms have been studied in numerous papers, see, e.g., Baur and Brandes [4].

Gansner and Koren [1] presented an approach to compute improved circular layouts for a given input graph $\mathscr{G} = (\mathscr{V}, \mathscr{E})$ in a three-step process. The first step computes a vertex order of $\mathscr{V}$ that aims to minimize the overall edge length of the drawing, the second step determines a crossing-free subset of edges that are drawn outside the circle to reduce edge crossings in the interior, and the third step introduces edge bundling to save ink and reduce clutter in the interior.



(a) One-sided layout



(b) Two-sided layout for $k = 1$

Figure 1: Circular graph layouts

Inspired by their approach we take a closer look at the second step of the above process, which, in other words, determines for a given cyclic vertex order an outerplane subgraph to be drawn outside the circle such that the remaining crossings of the chords are minimized. We generalize the problem from outerplane graphs to outer k-plane graphs, i.e., we ask for an edge set to be drawn outside the circle such that none of these edges has more than k crossings. For $k = 0$ this is the same problem considered by Gansner and Koren [1]. An example for $k = 1$ is shown in Figure 1b.

EXPERIMENTS/FUNDAMENTAL OF THE PROBLEM/EXAMINATIONS

We show how the previously described problem can be transformed into an equivalent problem on a more restricted graph class. The first tool we use is the circle graph $G = G_{\pi,\mathscr{G}} = (V, E)$ derived from $\mathscr{G}$ with an order π on the vertices. This graph has a vertex for every chord and two vertices $u, v \in V$ are connected, if the corresponding chords cross.

The problem of finding an optimal set of edges such that the exterior part has at most k crossings per edge can be modeled as a special case of a more general weighted version of the maximum constrained-degree subgraph problem.

Definition 1 *Given a weighted graph $G = (V,E)$ and $k \in \mathbb{N}$ find a set $V' \subset V$ such that the induced subgraph $G[V'] = (V',E')$ has $d(G[V']) \leq k$ and maximizes the weight*

$$W = W(G[V']) = \sum_{v \in V'} w(v) - \sum_{(u,v) \in E'} w(u,v).$$

Setting the weight of vertices to their degree and the edges to constant one or two will yield the correct version for our purpose. This is the case since we want to find a set of vertices which eliminate a lot of crossings in the original layout of $\mathscr{G}$ and the degree of a vertex $v \in V$ corresponds to the number of crossings of the original chord. The weight on the edges can be used to count the crossings moved to the exterior or not.

RESULTS AND DISCUSSION

The general problem as defined in Definition 1 is NP-hard making it unlikely to find a time-efficient algorithm solving it. For fixed k we present an algorithm running in $O((k\gamma)^{2k}\ell)$, where ℓ is the so called cord length of the circle graph, i.e. the sum of the length of all chords, γ is the maximum degree of the circle graph G and k is the maximum number of crossings allowed for an exterior edge. For $k = 0,1,2$ this yields a still practical usable algorithm for medium sized graphs.

The algorithm itself is an extension of the independent set algorithm for circle graphs presented by Valiente [2] and uses a dynamic programming approach. For $k = 1$ it can be described rather simple, once you observed that the set of chords can be split along sets of one single chord or two intersecting ones into independent parts.

CONCLUSION

In summary we showed how a crossing-optimal two-sided circular layout can be found for the case that each exterior edges can have at most k crossings. We prove that the problem is NP-hard for general k, but can be solved in polynomial time for fixed k.

For practical application of our algorithm we aim to integrate bundling of the edges into our algorithm and evaluate the visualization against other circular layouts, especially Ganser and Korens algorithm [1].

REFERENCES

[1] E. R. Gansner and Y. Koren. Improved circular layouts. In Graph Drawing (GD06), volume 4372 of LNCS, pages 386398. Springer, 2007.

[2] G. Valiente. A new simple algorithm for the maximum-weight independent set problem on circle graphs. In Algorithms and Computation (ISAAC03), volume 2906 of LNCS, pages 129137. Springer, 2003.

[3] S. Masuda, T. Kashiwabara, K. Nakajima, and T. Fujisawa. On the NP-completeness of a computer network layout problem. In Circuits and Systems (ISCAS87), pages 292295. IEEE, 1987.

[4] M. Baur and U. Brandes. Crossing reduction in circular layouts. In Graph-Theoretic Concepts in Computer Science (WG04), volume 3353 of LNCS, pages 332343. Springer Berlin Heidelberg, 2004.

[5] Purchase, Helen C., Robert F. Cohen, and Murray James. Validating graph drawing aesthetics. In International Symposium on Graph Drawing, pp. 435-446. Springer Berlin Heidelberg, 1995.