A CONTINUUM MICROMECHANICS APPROACH TO THE STRENGTH OF PLANAR FIBER NETWORKS: PAPER MATERIAL APPLICATIONS

Pedro Miguel Jesus de Sousa Godinho[a], Marina Jajcinovic[b], Leopold Wagner[a], Viktória Vass[a],
Wolfgang Johann Fischer[b], Thomas Karl Bader[c], Ulrich Hirn[b], Wolfgang Bauer[b],
Josef Eberhardsteiner[a], Christian Hellmich[a,*]

[a]E202 – Institute for Mechanics of Materials and Structures
[b]Graz University of Technology, Graz, Austria
[c]Linnaeus University, Växjö, Sweden

INTRODUCTION

The elastic properties of any micro-heterogeneous material depend on its inherent microstructure. One class of micro-heterogeneous materials are so-called fibrous materials. Their microstructure is made up of fibre networks, in which the individual fibres are connected via fibre-fibre bonds. In the majority of cases, the fibres are more or less parallel to one plane; one speaks of planar fibrous materials. Planar fibrous materials find numerous applications, which range from thermal and sound insulators, tissue templates, as well as gas and fluid fillers, to various paper product applications, including healthcare applications. One particularly widespread planar fibrous material is paper, a network of mechanically and/ or chemically treated wood fibres, so-called pulp fibres. In all the aforementioned applications, as well as in paper production, the mechanical properties, such as elasticity and strength of the planar networks, are of crucial importance. Therefore, it is not surprising that various mathematical models for the mechanical interaction of pulp fibres within the overall material "paper" have been proposed. However, none of these models explicitly accounted for the scale difference between the loads applied to the overall material and those acting on the level of the individual fibre. This motivated our research.

MICROMECHANICS-BASED LINEAR ELASTIC STRENGTH MODEL

In a first research stage, we filled the essential conceptual gap mentioned in the introduction with the development of a new micromechanics-based linear elastic model: We first recalled the fundamental micromechanical concept of the representative volume element and the corresponding stress and strain average rules, before we specified these rules for planar fibre networks such as paper material. Then we introduced elastic material behaviour at the fibre level, and derived so-called concentration relations for upscaling this behaviour to the planar network level. Combination of these relations with matrix-inclusion problems of the Eshelby-Laws type yielded closed-form semi-analytical expressions for the paper stiffness tensor, as a function of fibre stiffness and porosity. The self-consistent linear elastic model, which highlighted the importance of the fibre's anisotropy for the overall elastic behaviour, was confirmed by various multiscale experiments[1]. We here build upon our excellent results for the linear elasticity of planar networks to predict their strength. Given the self-consistent nature of our linear elastic model, stresses in single fibres are stresses in fibre-fibre bonds, and vice-versa. Therefore, we use a self-consistent linear elastic model-adapted Tresca-like function to connect the elastic limits of fibre-fibre bonds (that is of "single fibres") and planar networks. More specifically, we use the concentration relations derived from the linear elastic model, as well as the aforementioned Tresca-like function, to upscale 5-, 50-, and 95%-quantiles of a lognormally distributed sample of ultimate in-plane (mode II) shear strength of “unbeaten” unbleached softwood pulp fibre-fibre bonds to the yield in-plane uniaxial tensile strength of corresponding networks.

RESULTS AND DISCUSSION

Predictions based on the 50%-quantile almost perfectly agree with experiments on the yield in-plane uniaxial tensile strength of laboratory paper sheets of variable porosity (see Figure 1). These results emphasize the role of inter-fibre bond shear strength in the overall strength behaviour of the planar network and suggest that the presented linear elastic strength model may very well constitute an additional support tool in the design of paper production processes.

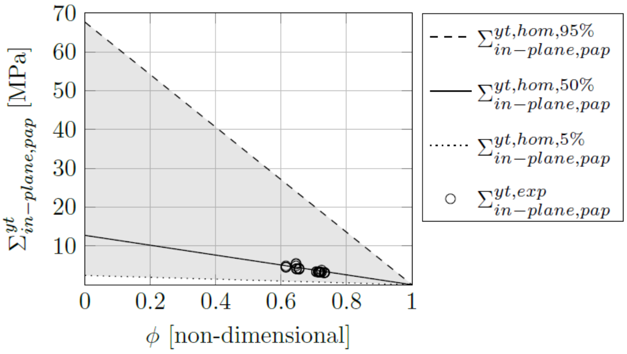

Figure 1: Experimental validation of micromechanics-based linear elastic strength model

CONCLUSION

The excellent agreement between our predictions and the corresponding experiments, strongly indicates that the elastic and strength properties of paper (and more generally of planar fibrous materials, such as thin films), can be accurately, micromechanically predicted from the elastic properties of the fibres leading to their formation, and from the strength properties of bonds between them – respectively. Future research efforts include the expansion of this model to other material classes and related symmetries, as well as to more complex mechanical phenomena.

REFERENCES

[1] P.M.J.S. Godinho, L. Wagner, V. Vass, J. Eberhardsteiner, C. Hellmich: A continuum micromechanics approach to the elasticity of planar fiber networks and its application to paper. In: “CD-ROM Proceedings of the World Conference on Timber Engineering (WCTE 2016)”, J. Eberhardsteiner, W. Winter, A. Fadai, M. Pöll (Eds.); published by: Vienna University of Technology; Grafisches Zentrum HTU GmbH, Vienna, 2016, ISBN: 978-3-903039-00-1 (2016), Paper-Nr. 1074.