AN EFFICIENT SIMULATION TECHNIQUE FOR THE EDDY CURRENT PROBLEM IN LAMINATED IRON CORES

Markus Schöbinger[a], Joachim Schöberl [a], Karl Hollaus [a]

[a]E101 - Institute of Analysis and Scientific Computing

INTRODUCTION

Laminated iron cores are an essential part of many electrical devices, for example transformers. Their property to be composed of a large number of thin iron laminates is essential to decrease the losses caused by eddy currents. From a numerical point of view this property poses a great challenge, since simulating the behavior of the iron core requires models incorporating each single laminate. In the classical finite element setting this causes the resulting equation system to become unfeasibly large. Therefore different methods have to be developed to calculate an approximated solution in a reasonable amount of time.

EXPERIMENTS/FUNDAMENTAL OF THE PROBLEM/EXAMINATIONS

The main topic of interest is the eddy current problem, which follows from the Maxwell equations. For the quasi-static magnetic field the weak formulation is given as: Find the magnetic vector potential $\vec{A}$ so that

$$\int_\Omega \mu^{-1}(\vec{A})\,\mathrm{rot}\vec{A}\,\mathrm{rot}\,\vec{v}\,d\Omega + \frac{\partial}{\partial t}\int_\Omega \sigma\vec{A}\vec{v}\,d\Omega = \int_\Omega \vec{J}\vec{v}\,d\Omega + \int_{\Gamma(\Omega)} \vec{K}\vec{v}\,d\Gamma$$

for every suitable test function $\vec{v}$ where μ is the magnetic permeability, σ the electric conductivity and the right hand side variables $\vec{J}$ and $\vec{K}$ allow the prescription of current densities in the volume and on the boundary of the domain, respectively.

For ease of presentation a simplified setting will be considered where μ is assumed to be independent of $\vec{A}$ (resulting in a linear equation) and the time harmonic solution is calculated, which allows to eliminate the time dependency by using complex numbers. Furthermore only a two dimensional cross section is considered and the problem is assumed to be independent of the third dimension. Prescribing only the boundary current, this leads to the two dimensional equation

$$\int_\Omega \mu^{-1}(\vec{A})\,\mathrm{rot}\vec{A}\,\mathrm{rot}\,\vec{v}\,d\Omega + i\omega\int_\Omega \sigma\vec{A}\vec{v}\,d\Omega = \int_{\Gamma(\Omega)} \vec{K}\vec{v}\,d\Gamma \qquad (1)$$

with the angular frequency ω and the imaginary unit i.

Figure 1 shows the solution in the iron domain of an academic example featuring only ten laminates. One can observe, that the solutions in each laminate admit little variance compared to each other. This encourages the idea that it might be sufficient to calculate the behavior of the solution in a single laminate and then propagate this

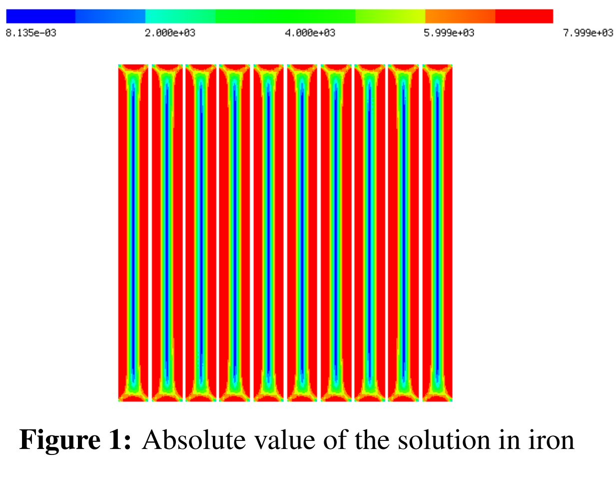

Figure 1: Absolute value of the solution in iron

result accordingly. These so called cell problems
give rise to many classical homogenization methods [1].

Here a slightly different ansatz will be used. The idea is to write the solution as a superposition of a mean value with local so called micro-shape functions to account for the behavior in each laminate. In the simplest form the ansatz

$$\vec{A} = \vec{A_0} + \varphi \begin{pmatrix} 0 \\ A_1 \end{pmatrix} + \nabla(\varphi w) \tag{2}$$

is used where the functions $\vec{A_0}$, A_1 and w are calculated on a much cheaper mesh which treats the iron core as a bulk. φ is the micro-shape function, which in this case is chosen to be a piecewise linear spline on each laminate. The approximation can be improved by adding additional micro-shape functions of higher order to (2) in a similar fashion. The ansatz (2) is then used in (1) together with averaging methods for the resulting coefficients to generate the new set of equations, as shown for example in [2].

RESULTS AND DISCUSSION

The presented method allows for the calculation of efficient approximation for the eddy current problem in laminated materials. In an example with one hundred laminates the meshes shown in figure 2 were used for calculations and for finding a reference solution to estimate the error. While the reference mesh has about 200 times more elements, the difference between the solutions measured in the L^2 norm is only 0.3%

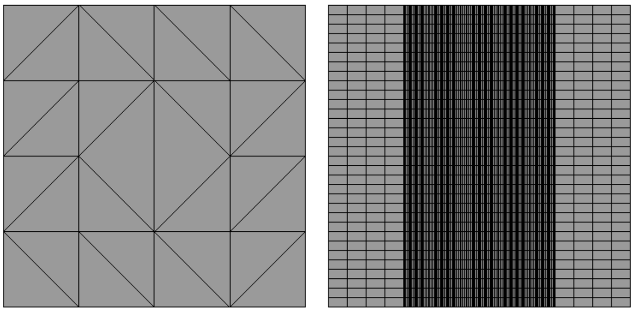

Figure 2: Cheap calculation mesh and reference mesh for 100 laminates

CONCLUSION

Simulating laminated materials using micro-shape functions has proven to be an efficient method for the linear eddy current problem. Future research will deal with generalizing this approach to the nonlinear case in the time domain.

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