TRANSFORMATIONS AND BASE SHAPE ANALYSIS

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INTRODUCTION

Principal aim of this topic is to investigate a novel approach to the base shape analysis of a given shape, that is, a decomposition of the shape into geometrically meaningful "base shapes" that provides geometric insight and hence lends itself to an intuitive refinement of a design while preserving geometric structure. This will greatly facilitate design processes that involve free-form shapes and transitions between physical and computational modelling.

FUNDAMENTAL OF THE PROBLEM

Darboux-Bäcklund-transformation and cross-ratio are two key points which I have had to become familiar with. Darboux-Bäcklund-transformations of curves are a kind of transformations which have helped to preserve the defining geometric features of their respective base surface classes [1]. They are considered a special type of Ribaucour transformation which is formed by two curves $x, \hat{x} : I \to \mathbb{R}^n$ if tangents at corresponding points $x(s)$ and $\hat{x}(s)$ are tangent to a common circle $c(s)$, i.e., if its tangent cross-ratio

$$cr = x'(\hat{x}-x)^{-1}\hat{x}'ds(\hat{x}-x)^{-1}$$

is real.

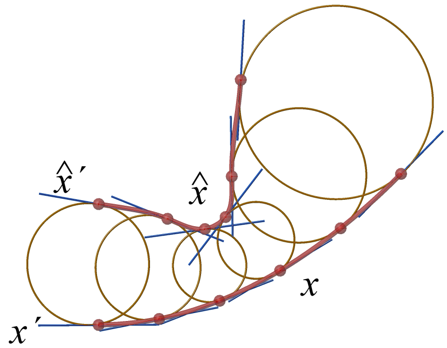

Figure 1: Darboux-Bäcklund-transformation

From three non-collinear points q_1, q_2, q_4 and a real number, there exists a fourth point on a circle such that

$$cr = (q_1 - q_2)(q_2 - q_3)^{-1}(q_3 - q_4)(q_4 - q_1)^{-1}$$

holds.

Thus, repeated transformations via cross-ratio generate new curves in the space.

We explore the relation between analogous smooth and discrete theories in a setting to analyze if the interplay between them becomes tangible.

From a given discrete curve, we analyze new Darboux transforms by observing how the cross-ratio, the initial point and the number of points in which the initial curve is divided affect them.

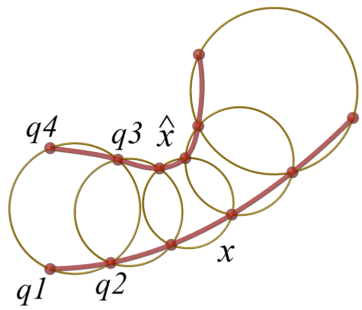

Figure 2: Cross-ratio-transformation

Our next mission is to compose some Möbius transformations [2] in order to get specific curves in the space. This is possible because the composition of two Möbius transformations is again a Möbius transformation [3].

$$z = f_2^{-1}(f_1(z)) = p_2$$

Interfacing areas between my research and the research fields of my institute are Transformation theory of curves and surfaces, Discrete differential geometry, Geometric modelling and Architectural geometry.

RESULTS AND DISCUSSION

From the same value of cross-ratio, the transformation of a smooth curve will be different from the same curve discretized. Likewise, two different discretizations will have different transformations. These differences will be null in transformations of the initial point and will increase as we move away from it.

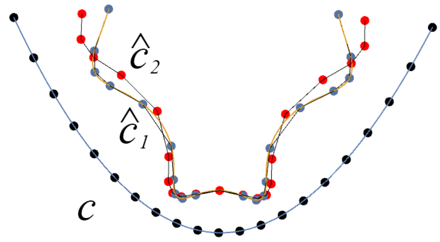

Figure 3: Smooth-discrete curves

Using Darboux-Bäcklund-transformation, it is possible to get any curve c from another one c' divided into the same number of segments if and only if c is transformed the same number of times than the number of segments have and the value of cross-ratio in every transformation will be prescribed for all the segments (r,s,t) belonging to that curve.

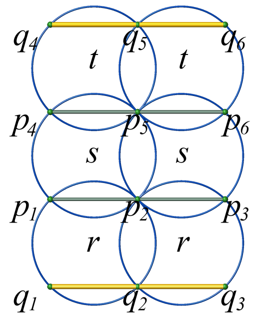

Figure 4: Composition Möbius transformations

In the same way, a curve c can be transformed being prescribed the initial and final points p_0, p_n of the transformation. The cross-ratio will be prescribed for all the segments, except for the last two ones $p_n, pn-1, p_{n-2}$, because cross-ratio in p_n, p_{n-1} and p_{n-1}, p_{n-2} shall ensure that p_{n-1} will coincide in both transformations.

CONCLUSION

As transformations are intimately related to the generation of discrete and semi-discrete surfaces, the control of developed methods will not only apply to smooth surfaces used in design, but also to faceted and panelled surfaces that lend themselves more easily to modelling and physical construction. The next steps will be to analyze the advantages over other forms of design [4].

REFERENCES

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